MATH 17 – QUIZ 6
8 NOVEMBER 2010

Name: SOLUTION

NO CALCULATORS

1. Determine whether the following series converge or diverge. If you can, find the value of the sum.

(a) $\sum_{n=1}^{\infty}\left(\frac{n+1}{n}\right)^n$

(b) $\sum_{n=1}^{\infty}\frac{5\cdot 2^n}{3^n}$

(c) $\sum_{n=0}^{\infty}\frac{1}{n^2+n+1}$

a) $\sum_{n=1}^{\infty}\left(\frac{n+1}{n}\right)^n$ diverges because $\lim_{n\to\infty}\left(\frac{n+1}{n}\right)^n = e \neq 0$.

(Test for divergence)

b) $\sum_{n=1}^{\infty}\frac{5\cdot 2^n}{3^n}$ is a geometric series with $r=\frac{2}{3}$. Since $|r|<1$, the series converges. Its value is $\frac{a}{1-r} = \frac{10/3}{1-\frac{2}{3}}$

$= \boxed{10}$

c) Compare $\sum_{n=0}^{\infty}\frac{1}{n^2+n+1}$ to $\sum_{n=1}^{\infty}\frac{1}{n^2}$.

$\sum_{n=1}^{\infty}\frac{1}{n^2}$ converges because it is a p-series with $p=2>1$.

For $n\geq 1$, $\frac{1}{n^2+n+1}\leq\frac{1}{n^2}$.

Thus, $\sum_{n=0}^{\infty}\frac{1}{n^2+n+1}$ converges also.